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TECHNICAL MEMORANDUM

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TITLE- Backup Technique for Determining AAP Cluster Orientation about Sun Line

TM- 70-1022-4

FILING CASE NO(S)- 620

DATE- March 19, 1970

AUTHOR(S)- R. A. Wenglarz

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(ASSIGNED BY AUTHOR(S))-

ABSTRACT

In its nominal attitude, the AAP Cluster geometric Z axis is directed towards the sun. The orientation about the Z axis is required in order to accurately acquire the local vertical attitude to conduct Earth resources experiments. Only a single star tracker is available to provide attitude reference for determining this orientation. No backup means for attitude reference is currently defined.

A method that uses measurements of CMG angular momentum components is herein formulated for determining the Cluster rotation angle about the Z axis. This method could provide a backup means for attitude reference in the event of star tracker failure. Preliminary analysis indicates this technique is feasible.

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955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

SUBJECT: Backup Technique for Determining AAP Cluster Orientation about Sun Line - Case 620

DATE: March 19, 1970

FROM: R. A. Wenglarz

TM 70-1022-4

TECHNICAL MEMORANDUM

1.0 INTRODUCTION

During normal operation of the AAP Cluster, the configuration geometric Z axis is directed towards the sun and the configuration orientation about the Z axis is such as to minimize the angular momentum accumulated by the control moment gyros (CMGs). This orientation can be determined by using the star tracker for attitude reference. However, in the event of a star tracker failure, no backup for determining the Cluster orientation about the Z axis has been defined.

Following is a proposed method for determining this rotation angle along with the outline of a preliminary analysis. This method utilizes measurements of CMG angular momentum components which reflect the external torques on the spacecraft which in turn reflect the spacecraft orientation.

2.0 SYSTEM DESCRIPTION

The configuration to be considered moves along a circular orbit and its position is described by the angle $\Omega_o t$ from orbital midnight. Orbital midnight is the point given by the dark side intersection of the orbit path with a plane perpendicular to the orbit which contains the centers of both the Earth and sun. Ω_o is the orbit rate and t is the time measured from the passing of orbital midnight. The angle between the line to the sun and the orbit plane is termed β.

The spacecraft is held by CMGs in a specified attitude characterized as follows: Let XYZ be a right-handed set of mutually orthogonal axes fixed in the spacecraft with origin at its mass center S and let $\underline{i}_1$, $\underline{i}_2$, $\underline{i}_3$ be unit vectors parallel to X, Y, Z respectively. The Z axis is directed along a line from the sun to the spacecraft and the orientation of the spacecraft about the Z axis is described by $-\varepsilon$, the angular rotation about the Z axis that rotates the X axis into the orbital plane. The elements of the spacecraft inertia matrix associated with XYZ are J_{nm}; n,m=1,2,3.

Let a second set of right-handed, mutually orthogonal axes X'Y'Z' with origin at S move along the orbit with the spacecraft, having X' along the instantaneous velocity vector, Y' normal to the orbit plane, and Z' along the local vertical line from the Earth center to the spacecraft. Unit vectors $\underline{n}_1$, $\underline{n}_2$, $\underline{n}_3$ are parallel to X', Y', Z' respectively.

3.0 DETERMINATION OF SPACECRAFT ATTITUDE

3.1 Only Gravity Gradient Torques Considered

For only gravity gradient torques considered, the method for determination of the rotation angle ε of the spacecraft about the Z axis is straightforward. Discussion of this method is an aid in presenting the more complex problem for which aerodynamic torques are also considered.

If aerodynamic torques are neglected, the equations of motion show in Appendix A (Equation (A-6)) that the change in some time interval of each component of CMG angular momentum is equal to the integral over that interval of the corresponding component of the spacecraft gravity gradient torque.

$$\Delta H_i = \int_{t_o}^{t} T_i^G d\tau \qquad i=1,2,3 \tag{1}$$

Defining

$$H_i^G = \int_{t_o}^{t} T_i^G d\tau \qquad i=1,2,3 \tag{2}$$

then Equation (2) and Equations (B-3)-(B-7) can be used to show that H_i^G is a function of orbit position $\Omega_o t$, initial time t_o, sun angle β, spacecraft inertia elements J_{nm}, n,m=1,2,3, and spacecraft rotation angle ε about the Z axis; that is

$$\Delta H_i = H_i^G(\Omega_o t, t_o, \beta, J_{nm}, \varepsilon) \qquad i=1,2,3 \tag{3}$$

For the AAP Cluster, $\Omega_o t$ and β can be made available from the ATM navigation program. J_{nm} and t_o are known quantities and the change ΔH_i in CMG angular momentum components can be measured so that the only unknown is ε. Then any one of the three Equations (3) can be solved for ε. For a small deviation $\Delta\varepsilon$ from a known value ε_o, Equations (3) can be linearized in $\Delta\varepsilon$ and have the form

$$\Delta H_i = H_{i0}(\Omega_o t, t_o, \beta, J_{nm}, \varepsilon_o) + \Delta\varepsilon H_{i1}(\Omega_o t, t_o, \beta, J_{nm}, \varepsilon_o) \tag{4}$$

so that

$$\Delta\varepsilon = \frac{\Delta H_i - H_{i0}}{H_{i1}} \qquad i=1,2,3 \tag{5}$$

It can be demonstrated that H_{i0} and H_{i1} have about the same magnitude. Then, Equation (4) shows that the variation in ΔH_i for small deviations $\Delta\varepsilon$ is only a few percent so that measurements of CMG angular momentum must be able to distinguish these small variations and the values of H_{i0} and H_{i1} must be accurately established.

Perhaps the greatest source of uncertainty of H_{i0} and H_{i1} is uncertainty in J_{nm}(n,m=1,2,3) the elements of the spacecraft inertia matrix. J_{nm} might be accurately determined from Equations (3) provided the star tracker is operational during the early part of the mission. Then ε is measurable and can be considered known in Equations (3) while J_{nm} are regarded as unknowns. Equations (B-5) show that Equation (3) is linear in the six quantities J_{nm} so that Equations (3) considered at two different values of $\Omega_o t$ give six linear equations which could be solved for the six unknowns J_{nm}.

3.2 Both Gravity Gradient and Aerodynamic Torques Considered

Equation (A-6) relates the CMG angular momentum components to gravity gradient and aerodynamic torques.

$$\Delta H_i = \int_{t_o}^{t} T_i^G d\tau + \int_{t_o}^{t} T_i^A d\tau \qquad i=1,2,3 \tag{6}$$

Then, defining

$$H_i^A = \int_{t_o}^{t} T_i^A d\tau \qquad i=1,2,3 \tag{7}$$

and using the definitions of Equations (2)

$$\Delta H_i = H_i^G + H_i^A \qquad i=1,2,3 \tag{8}$$

Since aerodynamic torques are the least accurately predicted, the three Equations (8) might be considered to involve four unknowns, H_i^A, i=1,2,3, and ε. However, if the resultant aerodynamic force on the spacecraft is assumed opposite in direction to the spacecraft velocity vector, the three H_i^A are not independent and an additional equation can be determined and solved together with Equations (8) for the spacecraft rotation angle about the Z axis.

The three aerodynamic torque components T_i^A, i=1,2,3 given by Equations (B-12) and (B-11) are not independent of each other and it can be shown that

$$T_1^A(s\beta s\varepsilon s\Omega_o t - c\varepsilon c\Omega_o t) + T_2^A(s\beta c\varepsilon s\Omega_o t + s\varepsilon c\Omega_o t) + T_3^A c\beta s\Omega_o t = 0 \tag{9}$$

and for $s\Omega_o t \neq 0$,

$$T_3^A = T_1^A(-s\beta s\varepsilon + c\Omega_o t c\varepsilon / s\Omega_o t)/c\beta - T_2^A(s\beta c\varepsilon + c\Omega_o t s\varepsilon / s\Omega_o t)/c\beta \tag{10}$$

Integrating both sides of Equation (10), using integration by parts on terms involving $c\Omega_o t / s\Omega_o t$, and recalling the definitions in Equations (7) gives

$$H_3^A = \left[-s\beta s\varepsilon H_1^A - s\beta c\varepsilon H_2^A + (c\varepsilon H_1^A - s\varepsilon H_2^A) c\Omega_o\tau / s\Omega_o\tau \,\Big|_{t_o}^{t} + \Omega_o \int_{t_o}^{t} \frac{(c\varepsilon H_1^A - s\varepsilon H_2^A)}{s^2\Omega_o\tau} d\tau \right] \Big/ c\beta \qquad (11)$$

From Equation (7), $H_i^A(t_o) = 0$ so that

$$H_3^A = \left[H_1^A (c\varepsilon c\Omega_o t / s\Omega_o t - s\beta s\varepsilon) - H_2^A (s\varepsilon c\Omega_o t / s\Omega_o t + s\beta c\varepsilon) + \Omega_o \int_{t_o}^{t} \frac{(c\varepsilon H_1^A - s\varepsilon H_2^A) d\tau}{s^2\Omega_o\tau} \right] \Big/ c\beta \qquad (12)$$

Solution of the first two of Equations (8) for H_1^A and H_2^A and substitution of the result into Equation (12) gives

$$H_3^A = \left[(\Delta H_1 - H_1^G)(c\varepsilon c\Omega_o t / s\Omega_o t - s\beta s\varepsilon) - (\Delta H_2 - H_2^G)(s\varepsilon c\Omega_o t / s\Omega_o t + s\beta c\varepsilon) + \Omega_o \int_{t_o}^{t} \frac{[c\varepsilon(\Delta H_1 - H_1^G) - s\varepsilon(\Delta H_2 - H_2^G)] d\tau}{s^2\Omega_o\tau} \right] \Big/ c\beta \qquad (13)$$

Placement of H_3^A from Equation (13) into the third of Equation (8) yields an equation of the form

$$\Delta H_3 = H_3^G(\Omega_o t, t_o, \beta, J_{nm}, \varepsilon) + H_3^A(\Delta H_1, \Delta H_2, \Omega_o t, t_o, \beta, J_{nm}, \varepsilon) \qquad (14)$$

Now the functional form of H_3^A is known as determined by Equation (13) and the functional forms of H_i^G are known as determined by Equations (B-2)-(B-7). For the AAP missions, all the variables except ε in Equation (14) are known, measurable, or can be made available from the ATM navigation program. Consequently, Equation (14) can be solved for ε.

For a small deviation $\Delta\varepsilon$ from a known value ε_o, Equation (14) can be linearized in $\Delta\varepsilon$ and has the form

$$\Delta H_3 = H_{30}^G + \Delta\varepsilon H_{31}^G + H_{30}^A + \Delta\varepsilon H_{31}^A \tag{15}$$

so that

$$\Delta\varepsilon = \frac{\Delta H_3 - H_{30}^G - H_{30}^A}{H_{31}^G + H_{31}^A} \tag{16}$$

Again, it may be desirable to calculate values of J_{nm} during early parts of the mission when the star tracker is available. Then ε is known and Equation (14) can be taken at six different values of $\Omega_o t$ to provide six linear equations which could be solved for the six unknowns J_{nm}.

4.0 SUMMARY AND CONCLUSIONS

For the AAP Cluster oriented with the geometric Z axis towards the sun, a technique has been suggested for determination of the rotation angle ε of the cluster about the Z axis and a preliminary analysis has been outlined. Determination of ε involves measurements of the components of CMG angular momentum over a time interval. Two limitations of the method are that either on-board or ground-based integrations of CMG angular momentum components are required as demonstrated by Equation (13) and that the time interval of measurements and computations does not include either orbital midnight or orbital noon for which $s\Omega_o t=0$. For these times, the transition between Equations (9) and (10) is not legitimate and the integral of Equation (13) blows up.

Although the method for determining spacecraft orientation has been shown feasible, a detailed error analysis is in order. Possible sources of error result from extraneous torques, inaccuracies in measurement of CMG angular momentum components, and from the spacecraft not having a truly inertially fixed attitude so that Equation (A-1) is not strictly correct.

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Attachments
  Appendices A, B

APPENDIX A

EQUATIONS OF MOTION

The equations of motion are to be formulated for the spacecraft held with its Z axis towards the sun and with a constant deflection ε about the Z axis. While this orientation is not truly inertially fixed due to the motion of the spacecraft about the Earth, the motion of the Earth about the sun, and orbital regression, the angular rates associated with these motions are very small and herein will be considered negligible. Then, the angular velocity $\underline{\omega}^S$ of the spacecraft is considered zero.

$$\underline{\omega}^S = \underline{0} \tag{A-1}$$

The total angular momentum $\underline{H}^T$ is made up of the spacecraft angular momentum $\underline{H}^S$ and the CMG angular momentum $\underline{H}$.

$$\underline{H}^T = \underline{H}^S + \underline{H} \tag{A-2}$$

The time rate of change of the total angular momentum is equal to the sum of the gravity gradient torque $\underline{T}^G$ and aerodynamic torque $\underline{T}^A$, for all other torques assumed negligible.

$$\frac{d}{dt} (\underline{H}^S+\underline{H}) = \underline{T}^G + \underline{T}^A \tag{A-3}$$

and by Equation (A-1)

$$\frac{d}{dt} \underline{H}^S = \underline{0} \tag{A-4}$$

so that

$$\underline{H} = \underline{H}_o + \int_{t_o}^{t} \underline{T}^G d\tau + \int_{t_o}^{t} \underline{T}^A d\tau \tag{A-5}$$

where $\underline{H}_o$ is the value of $\underline{H}$ at time $t=t_o$. Writing in terms of components in the directions of the spacecraft geometric axis XYZ,

$$\Delta H_i = H_i - H_{io} = \int_{t_o}^{t} T_i{}^G d\tau + \int_{t_o}^{t} T_i{}^A d\tau \tag{A-6}$$

APPENDIX B

TORQUES

For purposes of eventually expressing environmental torques in terms of components in the directions of the vehicle geometric axes XYZ, the unit vectors $\underline{n}_2$ and $\underline{n}_3$ associated with orbital axes Y' and Z' are expressed in terms of $\underline{j}_1$, $\underline{j}_2$, $\underline{j}_3$.

$$\underline{n}_2 = s\varepsilon c\beta \underline{j}_1 + c\varepsilon c\beta \underline{j}_2 - s\beta \underline{j}_3 \tag{B-1}$$

$$\underline{n}_3 = (c\varepsilon s\Omega_o t + s\varepsilon s\beta c\Omega_o t)\underline{j}_1 + (-s\varepsilon s\Omega_o t + c\varepsilon s\beta c\Omega_o t)\underline{j}_2 + c\beta c\Omega_o t \underline{j}_3 \tag{B-2}$$

where s and c preceding ε, β, $\Omega_o t$ indicate sines and cosines of these quantities. Equation (B-2) may be rewritten

$$\underline{n}_3 = r_x \underline{j}_1 + r_y \underline{j}_2 + r_z \underline{j}_3 \tag{B-3}$$

where

$$r_x = c\varepsilon s\Omega_o t + s\varepsilon s\beta c\Omega_o t, \quad r_y = -s\varepsilon s\Omega_o t + c\varepsilon s\beta c\Omega_o t, \quad r_z = c\beta c\Omega_o t \tag{B-4}$$

Gravity Gradient Torque

The gravity gradient torque $\underline{T}^G$ acting on the vehicle can be written in matrix form

$$\underline{T}^G = 3\Omega_o^{\,2}\, \tilde{r} J \underline{r} \tag{B-5}$$

where J is the spacecraft inertia matrix. In terms of components in the directions of the vehicle geometric axes XYZ,

$$\underline{T}^G = T_1{}^G \underline{i}_1 + T_2{}^G \underline{i}_2 + T_3{}^G \underline{i}_3 \tag{B-6}$$

where $T_i{}^G$ components are computed by using in Equation (B-5)

$$\underline{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}, \qquad \tilde{r} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \tag{B-7}$$

with r_x, r_y, r_z given by Equation (B-4) and inertia matrix J with elements J_{nm} associated with XYZ.

Aerodynamic Torque

Since prediction with any great precision of torques resulting from aerodynamic drag forces does not appear likely, representation of aerodynamic torques will be kept as general as possible. The only assumption is that the resultant drag force on the spacecraft is in the opposite direction to the spacecraft velocity vector.

Consider in Figure 1 the intersection of the resultant aerodynamic force $\underline{F}^A$ with a plane passing through the spacecraft mass center S and perpendicular to $\underline{F}^A$. For the orbiting reference frame X', Y', Z', that intersection is designated by the position vector $\underline{p}$

$$\underline{p} = y\underline{n}_2 + z\underline{n}_3 \tag{B-8}$$

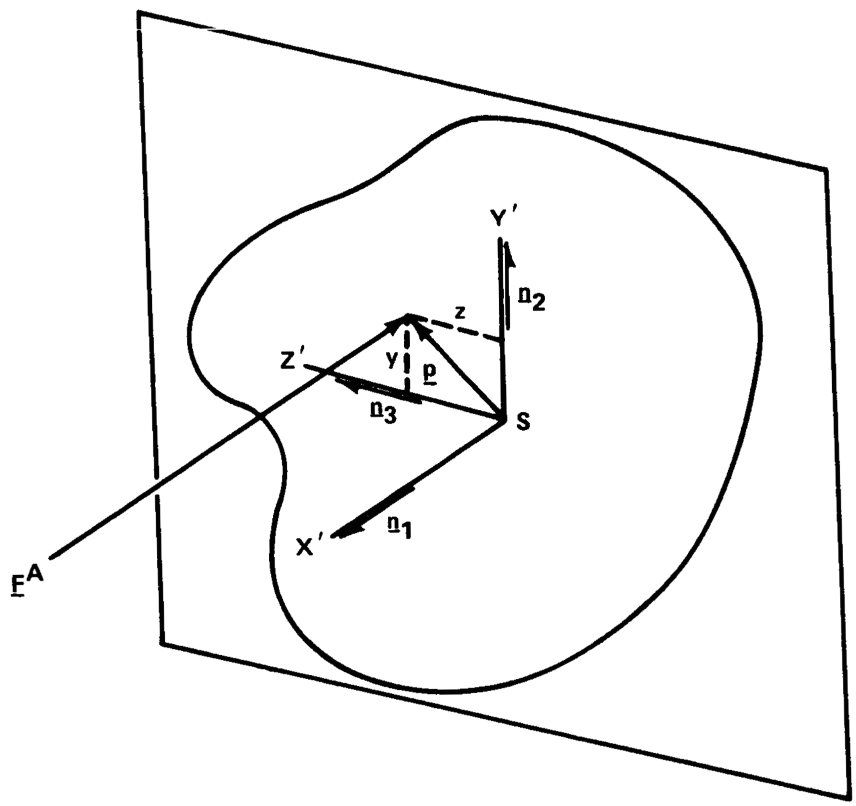

FIGURE 1 - AERODYNAMIC FORCE ON SPACECRAFT

as shown in Figure 1 and since the aerodynamic force is considered opposite in direction to spacecraft instantaneous velocity vector,

$$\underline{F}^A = -F^A \underline{n}_1 \tag{B-9}$$

where F^A is the magnitude of $\underline{F}^A$. Because the drag force is unspecified except for direction, its magnitude F^A and the position coordinates y and z of its point of application are considered to vary in an unknown manner.

The aerodynamic torque is given by

$$\underline{T}^A = \underline{p} \times \underline{F}^A = F^A(-z\underline{n}_2 + y\underline{n}_3) \tag{B-10}$$

and use of Equations (B-1) and (B-2) gives

$$\begin{aligned}\underline{T}^A = {} & F^A[yc\varepsilon s\Omega_o t + s\varepsilon(-zc\beta + ys\beta c\Omega_o t)]\ \underline{j}_1 \\ & + F^A\ [-ys\varepsilon s\Omega_o t + c\varepsilon(-zc\beta + ys\beta c\Omega_o t)]\ \underline{j}_2 \\ & + F^A\ (zs\beta + yc\beta c\Omega_o t)\ \underline{j}_3\end{aligned} \tag{B-11}$$

or

$$\underline{T}^A = T_1{}^A \underline{j}_1 + T_2{}^A \underline{j}_2 + T_3{}^A \underline{j}_3 \tag{B-12}$$

where $T_1{}^A$, i=1,2,3 are obtained by comparison of Equation (B-12) with Equation (B-11). (Note that knowledge of y and z is not required in the determination of rotation angle ε as described in Section 3.2.)